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Non-standard peak values of the Bose-Einstein correlations and their possible interpretation by a metric description of strong interactions

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Abstract. We critically reanalyze some recent experimental data on the Bose-Einstein (BE) correlations in pion production. We show that there is, in some cases, an experimental evidence for a peak height of the correlation function greater than two, contrarily to the predictions of the "canonical" theory of BE correlations. Although an explanation of such an "anomalous" value can be given by means of suitable phenomenological models, we show that this result is a straightforward consequence of the treatment of BE correlations within the framework of a description of strong interactions in terms of a deformed Minkowski metric.

1 Introduction

It is known since 1959 that the "normal" pionic correlations have an enhancement at low dipion mass M [1]. These "normal" pionic correlations are defined by

$$C_{(2)}^{\mathrm{nor}}(p_1,p_2)=\frac{N_U}{N_L}\frac{D_L(p_1,p_2)}{D_U(p_1,p_2)}, \qquad (1)$$

where $D_L(p_1,p_2)$ and $D_U(p_1,p_2)$ are respectively the probability density for like ($2\pi^+$ and $2\pi^-$) and unlike ($\pi^+\pi^-$) charged pionic pairs having four-momenta p_1 and p_2, and N_L and N_U are the total number of like and unlike combinations.

This effect is interpreted to be due to the fact that pions obey the Bose-Einstein statistics [2]. Afterwards, the works of several authors developed this interpretation and led to the formulation of the "canonical" theory of Bose-Einstein correlations (CTBEC) [3]. In this theory, the pionic correlations are defined by

$$C_{(2)}^{\mathrm{th}}(p_1,p_2)=\frac{D_L(p_1,p_2)}{D_0(p_1,p_2)}, \qquad (2)$$

where $D_0(p_1,p_2)$ is the probability one would have in absence of correlation. They are related to the square of the Fourier transform of the pionic source distribution $F_{(2)}(p_1,p_2)$ by

$$C_{(2)}^{\mathrm{th}}(p_1,p_2)=N[1+\lambda F_{(2)}(p_1,p_2)],$$

where N is a normalization factor and λ, called "incoherence parameter", is bounded by

$$0\le\lambda\le 1. \qquad (3)$$

The CTBEC predicts $\lambda=0$ if the pions are produced coherently and $\lambda=1$ in case of total incoherence.

The interpretation given by the CTBEC has been disproved by recent analyses [4–6] which studied the "normal" correlations in $\bar{\mathrm{p}}\mathrm{N}$ annihilations at rest as a function of the four-momentum transfer[1]

$$Q=\sqrt{(p_1-p_2)^2}=\sqrt{M^2-4m_\pi^2}.$$

Firstly, the CPLEAR collaboration studied the "normal" correlations in $\bar{\mathrm{p}}\mathrm{p}$ annihilation into four charged pions [4]. The results were that the peak heights are $h_{(2)}^{\mathrm{nor}}=C_{(2)}^{\mathrm{nor}}(0)=N(1+\lambda)>3$. This does not fit with the limit (3) because the data show a normalization factor N close to one.

Afterwards, one of us (M.G.) studied the "normal" correlations in the annihilations at rest [5, 6]

$$\bar{\mathrm{p}}\mathrm{n}\to 2\pi^+3\pi^-, \qquad (4)$$
$$\bar{\mathrm{p}}\mathrm{n}\to \pi^+2\pi^-, \qquad (5)$$

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[1] Actually, the paper [5] reported the $2\pi^+$ and $2\pi^-$ correlations as functions of Q^2. The correlations of the same reaction as functions of Q can be found in [7]

Table 1. Experimental peak heights. In the case of the second reaction, we give for $h^{\rm corr}_{(2)}$ the interval between the lower limit 1.4, corresponding to the case $f = 0.5$ at 95% CL, and the upper limit 3.1, corresponding to the case $f = 0.75$ at 95% CL

Reaction	$\bar{\rm p}{\rm n} \to \pi^+ 2\pi^-$	$\bar{\rm p}{\rm p} \to 2\pi^+ 2\pi^-$	$\bar{\rm p}{\rm n} \to 2\pi^+ 3\pi^-$
Reference	[6]	[4]	reevaluated
N_U/N_L	2	2	3/2
Q (GeV)	$0.00 \div 0.15$	$0.04 \div 0.10$	$0.00 \div 0.10$
$h^{\rm nor}_{(2)}$	6.89 ± 1.42	3.24 ± 0.29	4.20 ± 0.73
$f = N_B/N_U$	1/2	$0.50 \div 0.75$	2/3
$h^{\rm corr}_{(2)}$	3.44 ± 0.71	$1.4 \div 3.1$	2.80 ± 0.49

These analyses found again a peak height $h^{\rm nor}_{(2)} > 3$. In addition, they found that the peak height is generated by two different mechanisms: one is the Bose-Einstein symmetrization of the decay amplitude, which is responsible for a peak height $h^{\rm nor}_{(2)} \sim 2N$, the second is the positive interference between the like charged pions produced by the decay of $\pi^+\pi^-$ states having $I = 0$ and $I = 1$, which gives a further contribution $\Delta h^{\rm nor}_{(2)} \sim N$.

In the mean time, the other two of us (F.C. and R.M) developed a new interpretation of the BE correlations based on the assumption that the strong interactions are not isotropic, and gave a description of the BE phenomenon in terms of a deformed spacetime [8]. One consequence of this theory is that the maximum allowed value of the correlation peak is $h^{\rm th}_{(2)} = C^{\rm th}_{(2)}(0) = 3$. For this reason, we have analysed the experimental data of the Bose-Einstein correlations, in order to see if they are in agreement with such a prediction.

The content of the paper is as follows. In Sect. 2, we carry out an analysis of the existing experimental data, and show that the limit (3) for λ is indeed overcome. In Sect. 3, we discuss such a result from a theoretical point of view, showing that it is consistent with the predictions of the model of the BE phenomenon based on an anisotropic deformation of the Minkowski metric. Section 4 concludes the paper.

2 Comparison with the experimental data

Let us discuss the "anomalous" experimental data reported in papers [4–6].

The prediction on the correlation peak $h^{\rm th}_{(2)}$, based on the theory of [8], refers to the correlations one has when all the like charged pions are generated inside a small volume and in a small time interval ("fireball"). This does not occur when there are pions generated by the decay of long living mesons. Therefore, to our aims, we cannot take into consideration the results obtained using final states with π^0's, because there some charged pions are generated by the decay of the long living particles η, ω, η', etc. Thus, we are forced to use only the few data coming from the studies of final states made of only charged pions. These states are the two reactions (5) and (4) studied by one of us [5, 6] and one of the reactions studied by the CPLEAR collaboration [4], the annihilation at rest

$$\bar{\rm p}{\rm p} \to 2\pi^+ 2\pi^- . \qquad (6)$$

We report in the fifth row of Table 1 the correlation heights $h^{\rm nor}_{(2)}$ of these reactions, averaged in the interval shown in the fourth row of the same table. The value reported for the reaction (5) is that of the first bin in Fig. 2 of [6]. The height reported for the reaction (4) is the value obtained by summing the $2\pi^+$ and $2\pi^-$ distributions. It has been reevaluated by using the same data used in the previous analysis [5, 7].

To obtain the value of reaction (6), we use the correlations shown by Fig. 3 of the CPLEAR paper [4]. Here the data of the first bins have a high error. Therefore, to reduce the error, we use the weighted mean (WM) of the first three bins, $h^{\rm WM}_{(2)} = \overline{C}_i = 3.22 \pm 0.29$.[2]

It is well known that the "normal" correlations (1) are not a good approximation of the theoretical correlations (2), because the unlike distribution is the sum of three terms

$$D_U(Q) = D_R(Q) + D_B(Q) + D_I(Q). \qquad (7)$$

Here $D_R(Q)$ is the contribution due to the production of $\pi^+\pi^-$ resonances, $D_B(Q)$ is the contribution of the combinatorial background, i.e. of the non-interacting $\pi^+\pi^-$ pairs, and $D_I(Q)$ is the contribution of the interference between resonances and background.

A more correct approximation of the theoretical correlations can be obtained by using in the denominator the background distribution

$$C^{\rm corr}_{(2)}(Q) = \frac{N_B}{N_L}\frac{D_L(Q)}{D_B(Q)} = f(Q) C^{\rm nor}_{(2)}(Q).$$

Here N_B is the number of non-interacting $\pi^+\pi^-$ combinations, i.e. the integral of $D_B(Q)$, and $f(Q)$ is the function given by

$$f(Q) = \frac{N_B}{N_U}\left[1 + \frac{D_R(Q) + D_I(Q)}{D_B(Q)}\right].$$

Then, we evaluate the heights of the "corrected" correlations by making the following three hypotheses:

(i) The contribution of the resonances and of the interference is negligible for $Q \le 0.10$ GeV.
(ii) The integral of the interference distribution $D_I(Q)$ on the whole Q-interval vanishes.
(iii) The final state is generated by a succession of quasi-two body decays $S_n \to S_i + S_{n-i}$. (S_k indicates a state of k pions.)

The assumptions (i) and (ii) allow us to replace the function $f(Q)$ by the constant $f = N_B/N_U = 1 - N_R/N_U$,

[2] For completeness' sake, let us notice that weighted means could not work for quantities given by ratios (like the pionic correlation). So, we have performed also a logarithmic mean, which gives the result 3.24 ± 0.29, in good agreement with the WM value

where $N_R = N_U - N_R$ is the total number of $\pi^+\pi^-$ combinations which interact. In addition, the assumption (iii) fixes $f = 1/2$ for the reaction (5), because there one $\pi^+\pi^-$ pair does interact and the other is background.

We obtain the value of f for reaction (4) using the previous analysis made by one of us [9] which proved that it is dominated by the $f_0(1370)\pi^-$ channel, with negligible contamination by other channels. The $f_0(1370)$ is a resonance which decays $\rho^0\rho^0$ and $\sigma\sigma$ [9] (σ is the symbol for the 2π $I = 0$ S-wave interaction). Therefore, two $\pi^+\pi^-$ pairs interact and other four pairs are background in each event of this reaction. Then we have $f = 2/3$.

Reaction (4) can proceed through two kinds of channels: $T\pi$ and dd', where T denotes a 3π state, and d, d' denote 2π states. Therefore, f can range between 0.5 ($T\pi$ channel) and 0.75 (dd' channel). This yields the prediction

$$1.72 \pm 0.35 \leq h^{\mathrm{corr}}_{(2)} \leq 2.58 \pm 0.35. \tag{8}$$

The actual value of f cannot be estimated. In fact, the most recent analysis of reaction (6) (due to Diaz et al. [10]) dates back to the far-off 1970. This analysis interpreted the reaction as a mixture of 20% of an $I = 0$ initial state, dominated by the $A_2\pi$ channel, and of 80% of an $I = 1$ initial state, dominated by the $\rho^0 f$ channels, with f denoting an $I = 0$ dipion state (i.e. the σ or the $f_2(1270)$). Following such an analysis, one would have $f \sim 0.55$ and $h^{\mathrm{corr}}_{(2)} = 1.87 \pm 0.39$.

But the $I = 1$ initial state of reaction (6) is related by isospin invariance to the annihilation at rest

$$\bar{\mathrm{p}}\mathrm{n} \to \pi^+ 2\pi^- \pi^0. \tag{9}$$

Reaction (9) shows a relevant production of the three charged ρ states with relative ratios $\rho^+ : \rho^0 : \rho^- \sim 1 : 1 : 1$ [11]. This proves that this reaction is dominated by channels of the type $T\pi$, which predict exactly the ratios $\rho^+ : \rho^0 : \rho^- \sim 1 : 1 : 1$, while other possible channels, for example $\rho^-\rho^0$ or $\rho^-\sigma$, cannot produce any ρ^+. Such a conclusion is in complete disagreement with the results by Diáz et al. [10]. Therefore, the data of reaction (9) suggest that f in reaction (6) can be close to the value 0.75. If so, one would have, for reaction (6):

$$h^{\mathrm{corr}}_{(2)} = 2.58 \pm 0.53, \tag{10}$$

i.e. a value compatible with 3.

Indeed the actual peak height has to be higher, because the experimental values (reported in Table 1) have been obtained by a mean on an interval of Q. We estimate the amount of such corrections by making the assumptions that the background distribution $D_B(Q)$ is proportional to the phase space Q/M of the $\pi^+\pi^-$ system and that the correlation function is a Gaussian [2]

$$C^{\mathrm{corr}}_{(2)}(Q) = f[N + b\exp(-R^2Q^2)].$$

where N is the normalization parameter of the "normal" correlation, and $b = N\lambda$.

By using the fitted values of the parameters reported in [4–7, 12], we find that the correction factor to the correlation peak is about 1.05.

In addition, one has to remember that the correlation height can be deformed by the Coulomb interaction, which is attractive for $\pi^+\pi^-$ pairs and repulsive for like pion pairs. A common parametrization for this correction is [13]

$$\chi(Q) = \frac{e^{2\pi\eta} - 1}{1 - e^{-2\pi\eta}}, \quad \eta = \frac{\alpha m_\pi}{Q},$$

where $\alpha \sim 1/137$ is the fine-structure constant. This correction factor is $\chi = 1.066$ at $Q = 0.10\,\mathrm{GeV}$ and is higher at lower Q.

Then, we can conclude that, on account of the above considerations, the limits of the variability range of $h^{\mathrm{peak}}_{(2)}$ are actually higher than those given in (8). In particular, the upper limit (10) is expected to be close to 3.

3 Theoretical discussion

We want now to show that the non-standard peak values obtained by the analysis of the experimental data performed in the previous section find a natural explanation in the framework of a model of BE correlation, based on an anisotropic, deformed Minkowski space representation of strong interactions [4].

The main idea amounts to the point that strong interactions (at least in some cases, like the BE phenomena) may be nonpotential and/or nonlocal, and that such effects can be (in average) taken into account by considering a "deformed" spacetime inside the interaction region. Such a possible spacetime deformation inside hadrons was first considered, on a phenomenological basis, by Nielsen and Picek [14]. So, we assume that inside the interaction region where pions are produced (the "fireball" of the BE correlation) the space-time metric is no longer the usual, Minkowskian one, $g_{\mu\nu} = diag(-1, +1, +1, +1)$ but is instead given by the deformed, spatially anisotropic metric [8]

$$\eta_{\mu\nu} = diag(-b_0^2, +b_1^2, +b_2^2, +b_3^2) \tag{11}$$

($\mu, \nu = 0, 1, 2, 3$), where the metric parameters b_μ^2 depend on the energy of the process considered. Therefore, the generalized interval reads

$$ds^{\tilde{2}} \equiv dx * dx = -b_0^2c^2dt^2 + b_1^2dx^2 + b_2^2dy^2 + b_3^2dz^2. \tag{12}$$

Let us note that the spacetime described by the interval (12) actually has zero curvature, and therefore *it is not* a true Riemannian space (whence the term "deformation" used to describe such a situation). Therefore, on this respect,thepresent description ofstrong interactions in terms of a metric change is different from that adopted in general relativity to describe gravitation. Moreover, as shown in [8], metric (11) reduces to the Minkowskian one, $g_{\mu\nu}$, for a suitable, characteristic value E_0 of the energy. But the energy of the process is fixed and cannot be changed at will. Thus, although, in principle, it would be possible to recover the Minkowski space by a suitable change of coordinates (e.g. by a rescaling), this would amount to a mere mathematical operation, devoid of physical meaning. On the other hand, the phenomenological analysis of [8], based on a fit to the UA1 experimental data, shows that the metric corresponding to the BE phenomenon is indeed deformed. Therefore the Minkowski space, in this framework, is a *physical*, and not a mathematical limit[3].

The spatial part of the nonlocal metric tensor η describes the spatial deformation of the boson source, whereas the time-parameter b_0 has the meaning of *time-correlation* (i.e. phase correlation) of the bosons. Precisely, the physical parameters a_μ of the fireball are related to the b_μ of η by the relations [8]

$$a_k = \hbar c b_k; \quad a_0 = \hbar b_0.$$

Moreover, in order to account for a possible anisotropic distribution of the boson subsources inside the total source, one has to take a four-vector source function, defined as [8]

$$\tilde{\rho}_\mu(r) = \frac{1}{4} a_\mu^4 e^{-\frac{r^2 a_\mu^2}{2}}. \tag{13}$$

The deformation (12) of the Minkowski metric induces a change in the phase factors of the boson wave function, essentially due to the deformation $*$ of the scalar product. Thus, the deformed symmetrized boson wave function reads [8]

$$\tilde{\psi}_{1\,2}^{BE}(x_1, x_2; r_1, r_2) = \frac{1}{\sqrt{2}} \left\{ e^{ip_1*(x_1-r_1)} e^{ip_2*(x_2-r_2)} + e^{ip_1*(x_1-r_2)} e^{ip_2*(x_2-r_2)} \right\}. \tag{14}$$

The intrinsic anisotropy of the strong interactions, expressed by (11) and (13), reflects itself in the fact that now *the correlation probability is different for different space-time directions*. This leads therefore to the following anisotropic form of the probability $D_L(p_1, p_2)$:[4]

$$D_L(p_1, p_2)(\mu) = \int \left| \tilde{\psi}_{1\,2}^{BE}(x_1, x_2; r_1, r_2) \right|^2 \times \rho_\mu(r_1) \rho_\mu(r_2) d^4 r_1 d^4 r_2. \tag{15}$$

Replacing (14) in the above expression of D_L, one gets, by calculations similar to the standard ones, the following expression for the second-order BE correlation function, $\tilde{C}_{(2)}$:

$$\tilde{C}_{(2)}(\mu) = 1 + |\tilde{F}_\mu|^2 \tag{16}$$

namely, a different correlation function for every space-time direction. According to (15), $\tilde{F}_\mu$ is essentially the generalized Fourier transform of the anisotropic distribution function of the sources inside the interaction region.[5]

Since, in the experiments, we measure a *global* BE correlation function (1), in order to compare the theoretical predictions of the anisotropic model with the experimental data, we have to suitably average (16) on all space-time directions. This can be done by assuming that the squared norm in (16) is (the absolute value of) a relativistic norm, built up from the spacetime metric. There are essentially two ways of doing this, namely, we can choose either the usual Minkowski metric $g_{\mu\nu}$, or the deformed one, $\eta_{\mu\nu}$. By assuming a Gaussian distribution for the sources, different for each spacetime direction [8], we have, in the former case:

$$\tilde{C}_{(2)}^{is} \equiv 1 + F_\mu^* g^{\mu\nu} F_\nu = 1 - e^{-\tilde{Q}^2/a_0^2} + \sum_{k=1}^{3} e^{-\tilde{Q}^2/a_k^2} \tag{17}$$

where $Q = p_1 - p_2 = (q_0, \mathbf{q})$, $\tilde{Q}^2$ is the deformed norm of the momentum transfer

$$\tilde{Q}^2 = Q^\mu \eta_{\mu\nu} Q^\nu = -Q_0^2 b_0^2 + Q_1^2 b_1^2 + Q_2^2 b_2^2 + Q_3^2 b_3^2. \tag{18}$$

The upper-index "is" in (17) means that the average $\tilde{C}_{(2)}^{is}$ is an *isotropic* one: it has been performed *outside* the interaction region, where the spacetime has its usual Minkowskian structure. The particles involved in the BE process do keep memory of the nonlocal, anisotropic forces that produced them through the different form of the correlation function, and the presence of the deformed metric parameters in the exponential function (cf. (17) and (18)).

The other possibility is to average the anisotropic correlation function (16) *inside* the interaction region, i.e. using the deformed metric (11). We get

$$\tilde{C}_{(2)}^{an} \equiv 1 + F_\mu^* \eta^{\mu\nu} F_\nu = 1 - b_0^2 e^{-\tilde{Q}^2/a_0^2} + \sum_{k=1}^{3} b_k^2 e^{-\tilde{Q}^2/a_k^2} \tag{19}$$

where "an" now means "anisotropic". The average (19) corresponds to differently weighting each direction, according to the fact that the deformed metric (11) implies a "renormalization" of the lengths, different for each direction. The correlation function (19) has been tested directly by a fit to the UA1 experimental data, and it yields a fit as good as that obtained by the usual Gaussian correlation function [8]. Moreover, it provides information on the shape and size of the fireball, including its "time" extension τ. This last parameter can be interpreted as a "mean-life" of the fireball, and it can be shown that it is related

[3] Such a fact leads to assume that, actually, we are working in a five-dimensional space-time, where energy just plays the role of fifth dimension [15]. Indeed, it can be shown that this five-dimensional space is now a true Riemannian one, with non-zero curvature. See [15] for more details

[4] Here, obviously, the momenta p_i are those measured in the laboratory frame, i.e. in full Minkowskian conditions. The information on the deformation inside the fireball is entirely contained in the deformation of the inner product, which in turn reflects itself, in the expression of the correlation function, in the deformed norm of the momentum transfer (see below). Such an effect, whereby the particles do keep memory of the anisotropy of the forces which produced them, can be regarded as a kind of *hadronic Einstein-Podolsky-Rosen effect*

[5] It must be stressed that the results obtained by our approach for the correlation function are related not only to the anisotropy of the source distribution (cfr. (13)) but also to the deformation of the inner product in the wavefunction phase (see (14)). This is evident by the very form of (17), (19), where the exponentials do contain the *deformed* norm (18) of the momentum transfer

to the energy width of the correlation function (i.e. the energy range where the experimental correlation function is greater than one. We refer the reader to [8] for further details)[6]

What we want now to stress is that *the two averaged correlation sections functions predict different limits for the peak values.* Indeed, in the case of $\tilde{C}_{(2)}^{an}$, (19), it is easy to see that, in the Minkowskian limit (since every b_μ^2 is of the order of 1/3 [8])[7], the upper value of $\tilde{C}_{(2)}^{an}$ $(Q = 0)$ is

$$\tilde{C}_{(2)}^{an} = 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{1}{3} = 1.67. \quad (20)$$

In the case of the isotropic average, we get instead

$$\tilde{C}_{(2)}^{is} = 1 + 1 + 1 + 1 - 1 = 3. \quad (21)$$

Notice that this theoretical upper limit for the correlation peak height is here derived for the first time, because only the anisotropic case was considered in [8].

We can therefore conclude that, in this model, the peak value $h_{(2)}^{\rm peak}$ of the correlation function is expected to vary within the values

$$1.67 \leq h_{(2)}^{\rm th} \leq 3. \quad (22)$$

4 Conclusions

The conclusions we can draw by the above discussion are the following:

– (a) We still confirm that the "canonical" theory of the Bose-Einstein correlations [3] is in disagreement with the experimental data.

– (b) The model of Cardone and Mignani [8] predicts $1.67 \leq h_{(2)}^{\rm peak} \leq 3$. We have found that the experimental data are in agreement with this prediction. This is a proof in favour of the model of the asymmetry in strong interactions. Moreover, the standard treatment of the BE correlation, valid in the (electromagnetic) optical case, does no longer hold for strong interactions. Indeed, the behaviour of BE-like phenomena in hadronic processes is a consequence of the anisotropy of the strong interactions. Paradoxically, the effect is enhanced at low energy: the value 2, obtained by the "optical" treatment of BE correlation in pion production, is only a lower limit on the peak value.

– (c) The interpretation given by Gaspero [5–7] is that the correlation peak height is generated by two mechanisms: the Bose-Einstein statistics and the positive interference between the like pions generated by the decay of $\pi^+\pi^-$ states having isospin $I = 0$ and $I = 1$. As is well known, the isospin is a quantum number related to a rotation in an abstract space. This work suggests that isospin could perhaps be related to the anisotropy of the strong interactions.

[6] Furthermore, in [8] the explicit dependence of the metric parameters b_μ^2 on the energy was derived by using the data of the UA1 ramping run. In this sense, *the functions $b_\mu^2(E)$ provide an effective dynamical description of the strong interaction in terms of a deformation of the spacetime metric* (at least for the two-pion BE phenomenon). See [8]

[7] This is due to the fact that, before the deformation, the spatial shape of the fireball can be considered spherical, with unit radius, so that the mean squared values of the spatial parameters b_k^2 (which are related to the spatial sizes of the interaction region) are of the order 1/3. Moreover, since both the spatial deformation of the source and the appearance of the time parameter b_0^2 are to be ascribed to the same (non-local) effects, it is expected that b_0^2, too, is of the same order of magnitude of the spatial parameters

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